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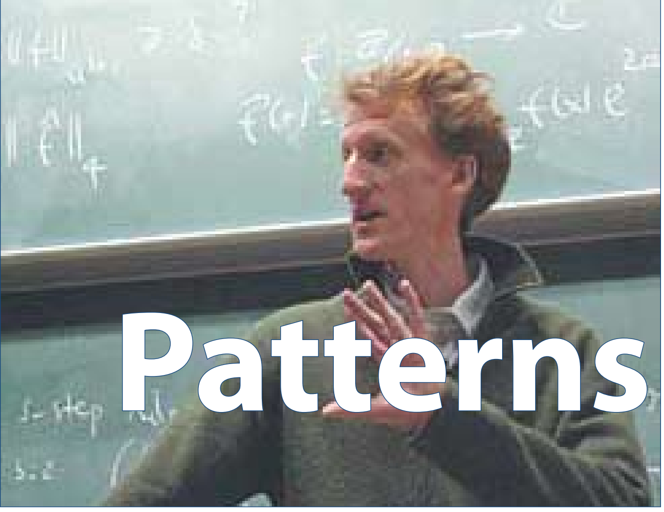

Patterns in prime numbers

Ben Green's introduction to Aotearoa was a shaky one; his first night was 4 September in Christchurch. Luckily he was staying in a motel where nothing even fell off the wall, with a Californian colleague who knew what to do in a quake. He spoke with Jenny Rankine.

Green is Herchel Smith Professor of Pure Mathematics at the University of Cambridge, and gave the London Mathematical Society's Forder Lectures throughout the country in September. He particularly enjoyed learning about Henry Forder, the Chair of Mathematics at the University of Auckland whose endowment founded this lectureship, and whose Euclidian geometry tied in with Green's work.

Green works in additive combinatorics, an area related to number theory, analysis and combinatorics. He is best known for the 2004 Green-Tao theorem, that there are infinitely many arithmetic progressions of prime numbers of any specified length.

These progressions are sequences of primes that differ by a constant amount; for example 5, 17, 29, 41 and 53 step up by 12; and 13, 43, 73 and 103 go up in jumps of 30. Before then, the largest known arithmetic progression had 22 primes.

Green and Australian mathematician Terence Tao started with a 1975 result by Hungarian mathematician Endre Szemerédi, who showed that in any infinite set of numbers that does not thin out too rapidly, there will be arithmetic progressions of all finite lengths.

However, primes do thin out rapidly. So Green and Tao cleverly pruned some non-primes; they generated a pseudorandom infinite number set containing primes and non-primes with few divisors for their size, for which Szemerédi's result still held.

Their 50-page, non-constructive existence proof did not include any arithmetic progressions of primes or say how to find them; it used a combination of ergodic theory (about mixing or averaging) and number theory.

Since then, Green has worked with collaborators to find out more about prime number patterns and understand the mathematics behind them. He and Israeli mathematician Tamar Ziegler have developed an asymptotic formula for the number of arithmetic progressions separated by particular numbers. "For example, if you want to know how many progressions of primes there are separated by 100, we can find out. We didn't do that for the results, but for the structures underlying them; generalizing Fourier analysis."

Green has also worked with Tao on finding slightly more exotic patterns of primes. "If d is the spacing, d can be a square number or a cube."

P1<.....>p1000

"The famous problems about prime numbers seem open - the Goldbach conjecture that every even number is the sum of two primes, and the Twin Prime conjecture, that there are infinitely many pairs of primes differing by two. My career goal might be to say something about one or the other. It's very hard now, there's no sensible way of attacking those questions."

Green won silver medals at the International Mathematical Olympiad in 1994 and 1995.

"Maths is unique as a discipline," he says, "in that being naive can actually be very helpful. These days if I think of an idea, I know of too many reasons why it can't work, so I tend to give up."

"I like everything about it - you can see all these interconnections, beauty, symmetry, surprising things. It's also very social; I haven't written a paper by myself for over seven years. I'm always emailing and meeting collaborators. I have a lot of fun doing it."

Crochet the hyperbolic plane

An exhibition of crochet in the hyperbolic shapes of various corals, called Seagardens Aotearoa, will open at the Estuary Arts gallery in Orewa from the end of November until the end of January. The shapes are based on the first easily usable physical models of hyperbolic space, developed by mathematician Daina Taimina in 1997, using ideas from William Thurston (see *IMAges* 8).

Crocheting these shapes is very simple; Seagardens Aotearoa co-ordinator Glenys Stace is working with local fibre arts groups to add to the display.

See also

www.seagardens.wordpress.com (the password for the museum page is museum)
www.math.cornell.edu/~dtaimina/